Squeezing Atomic Vibrations In An Optical Lattice. A New Mechanism Of Optical Cooling.

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We propose a new method to obtain a squeezed matter field of atomic vibrations by use of an optical lattice, and the laser pulse technique of Garrett et al used for acoustic phonons [1]. We show that it is possible to reduce the variance of atomic momentum to a value as low as the recoil momentum. Consequently, cooling to recoil energy can be achieved.

32.80.Pj,42.50.Lc,42.50.Vk

It this letter we propose and analyze the possible squeezing of a matter field comprising the atomic vibrations of an optical lattice (see e.g. Ref. [2]) which is created when atoms are spatially localized in the potential minima of a standing laser field at ultra-low temperature. As shown below this approach has major advantage compared to the phonon case [1]. For a realistic system, we show that significant squeezing can result, and that optical cooling can be achieved via this new method. An analogous but different method was recently proposed in Ref. [3] where it was assumed that the optical lattice is suddenly turned off and on. In our case, the background optical potential is always present but there exists an additional fast time-dependent component which leads to the squeezing of the phase distribution of atoms in the optical lattice.

It is known that the squeezed state [4] can be achieved for a Bose system with equally spaced oscillator levels, such as photons [5], polaritons [6], and phonons [6].

In the recent experiment [1], the squeezing of phonons in $KTaO_3$ has been achieved with a noise reduction of 10^{-4}. This small value is due to several factors limiting the squeezing that can be achieved by lattice phonons : phonon dispersion which leads to the decay of the induced coherent oscilations, residual anharmonicity (phonon-phonon interactions), finite pulse length of the incident laser pulse excitation. In the experiment [1] some compensation of these negative factors occured by tuning to a van Hove singularity in the two-phonon density of states. We were motivated to consider an optical lattice because the characteristic atomic vibration energy [7] is some 10^4 times less than for atoms in a vibration (phonon) mode in a "real" lattice. Additionally the phonons have essentially an Einsteinian (dispersionless) distribution. The method discussed here is thus free from the above limitations which leads to larger squeezing.

As a model of the optical lattice we consider a gas of two level atoms each of mass M and resonant energy $E_0 = \hbar\omega_0$, with transition dipole moment d and where the radiative width of the excited state is γ_R. The atoms of this gased are located in the minima of the potential relief formed by the external laser standing wave which can be characterized by the frequency Ω and the wave vector $k \approx \omega_0/c$. We assume that the detuning

$$\Delta\omega = \omega - \omega_0$$

is larger than the Rabi frequency Ω_R and the radiative width. The off-resonance conditions are required to avoid decay of the coherent state due to the presence of the excited state. For simplicity we consider a one dimensional model and the extension to three dimensions is straightforward because of the separation of the variables. The use of time-dependent off-resonant dipole potentials was shown to be successful for the manipulation of the atomic center of mass motion in optical lattices [8].

Consider first the states of the atom neglecting the radiative width in comparison with the detuning and the Rabi frequency. The energies of two eigen states can be found within the standard $u - v$ transformation for two levels. All our work is made in the rotating frame. Then for zero Rabi frequency the ground level has energy 0 and the excited level has complex energy $\Delta\omega + i\gamma_R$ which means that it has finite lifetime. For finite Rabi frequency Ω_R the hybridization of these levels must be taken into account. Since the Rabi-frequency is spatially dependent in the field of the standing wave we can represent the effective potential for the pseudoground state as [9]

$$U(x) \approx -\frac{\hbar\Omega_R^2}{2\Delta\omega} cos^2(kx) = -\frac{\hbar\Omega_R^2}{4\Delta\omega}(1 + cos(2kx)), \quad (1)$$

where k is the resonant wavevector. The last approximation implies an adiabatic condition, namely the vibrational motion within the optical lattice is much slower that the motion between two levels of a single atom. The frequency of the atomic vibrations near the potential minima in the effective potential (1) is given by

$$\omega_h = \sqrt{\frac{\Omega_R^2 \hbar (2k)^2}{4M\Delta\omega}}, \quad (2)$$

while the motion between the upper and lower levels is defined by $\Delta\omega$. The energy

$$E_R = \frac{\hbar^2 k^2}{2M} \tag{3}$$

corresponds to the atom having recoil momentum $q = \hbar k$ and represents the recoil energy E_R which is normally much less than the radiative width $\hbar\gamma_R$ and the detuning $\Delta\omega$ in the off-resonant case under consideration.

In analysing the optical lattice, we will assume the same type of excitation as in [1], namely a "delta" laser pulse detuned from resonance of the two level atoms. In the work [1], a rapid laser pulse, with duration $\tau \sim 70fs$ less than the inverse phonon Debye frequency was applied at $t = 0$, perturbing the system by the potential:

$$V(x) = -\alpha I x^2, \tag{4}$$

where x is the displacement, I is the intensity of the pulse and α is a proportionality factor which is unknown for a crystal and will be found below in the optical lattice. Taking into account the small value of pulse duration τ we can represent the time dependent perturbation by a δ function

$$V(x) = -\alpha I x^2 \tau \delta(t), \tag{5}$$

Linear terms in x are absent due to symmetry [10]. The system eigenfunction just after the perturbation ($t = 0^+$) is given as

$$\Psi(x, t \to 0^+) = \Psi(x, t \to 0^-) \cdot e^{i\xi M\omega_h x^2},$$
$$\xi = \frac{\alpha I \tau}{\omega_h M}, \tag{6}$$

We have introduced the dimensionless parameter ξ which characterizes the strength of the squeezing. In order to examine the amount of squeezing, we need to calculate the dispersions p^2 and x^2 in momentum and configuration space respectively. Following [1] we obtain

$$< p^2 >_{min} = < p^2 >_{0-} \frac{1}{2\xi^2 + 1 + 2\sqrt{\xi^4 + \xi^2}},$$
$$< p^2 >_{max} = < p^2 >_{0-} (2\xi^2 + 1 + 2\sqrt{\xi^4 + \xi^2}),$$
$$< x^2 >_{min,max} = < p^2 >_{min,max} /(M^2\omega_h^2), \tag{7}$$

Where $< p^2 >_{min}$ and $< p^2 >_{max}$ correspond to the maximum and minimum values of the momentum dispersions due to the vibrations of the optical lattice, and $< p^2 >_{0-}$ is the momentum dispersion of the atoms in the lattice before the external pulse given by Eq.(5) was applied to the system. The product of the minimum and maximum variances of p and x, respectively, remains the same as before switching on the external pulse. This conservation reflects the special properties of the harmonic oscillator potential.

For strong squeezing the result (7) can be simplified as

$$< p^2_{min} > \approx < p^2 >_{0-} \cdot \frac{E_i}{E_f} \tag{8}$$

where $E_{i,f}$ are the initial and final energies of the oscillatior, respectively.

Our goal is to estimate the minimum variance of momentum which can be achieved in the cooling scheme described above. This corresponds to the maximum value of squeezing for the atoms forming the optical lattice.

According to Eq.(8) the minimum width of the wave function in momentum space can be represented as $2ME_i^2/E_f$. Let the depth of the well created by the standing wave be U. The energy U defines the maximum energy which can be transmitted to the atom. The minimum initial energy E_i is just the level spacing for the oscillator-type equidistant levels near the bottom of the well:

$$\hbar\omega_h \sim \sqrt{E_R U}.$$

Taking into account all the above estimates we find that the minimum possible width in momentum space which can be reached for the optical lattice in this method is limited by the recoil momentum

$$< (\Delta p)^2 >_{infim} \approx (\hbar 2k)^2. \tag{9}$$

This is the maximum possible squeezing which can be obtained without spontaneous emission since $2k$ is the minimum momentum transfer for the interaction with the laser field. Below we will study whether this strong squeezing can be achieved for real conditions.

One should note that the anharmonicity might produce a further constraint for the width of the squeezed state. Additionally it gives rise to the slow decay of the amplitude of the variance oscillations. However under the current experimental conditions [8] this effect will change $< (\Delta p)^2 >_{infim}$ by a factor of the order of unity at least for the first vibration. A more detailed account of the influence of anharmonicity will be published separately [11].

The adiabatic treatment of the atomic motion in the potential of the laser field is valid automatically since the detuning $\Delta\omega$ is assumed to be much greater than both the recoil energy and the energy corresponding to the Rabi frequency.

In the off-resonant case $\Omega_R \ll \Delta\omega$ the decay rate of the g-state can be written as (see e.g. Ref. [12])

$$\gamma_g \approx \gamma_R \frac{\Omega_R^2}{\Delta\omega^2}. \tag{10}$$

Any decay event leads to a change of the phase of the atom wave-function as well as the momentum of the atom. The loss of coherence might occur and squeezing will be hard to achieve. That is why we have to avoid spontaneous emission. Since squeezing occurs during the period of the atom vibration, defined by its inverse frequency $(\omega_h)^{-1}$, this frequency should be much larger then the decay rate (10)

$$\gamma_g \ll \omega_h. \quad (11)$$

We will consider the case of two fast waves counter-propagating, so:

$$E(x,t) = E_o(cos(\omega_*(t-x/c))\varphi(\frac{t-x/c}{\tau}) + \\ +cos(\omega_*(t+x/c))\varphi(\frac{t+x/c}{\tau})), \quad (12)$$

where τ is the duration of the pulse, the function $\varphi(a)$ decreases rapidly for $a > 1$ and for detailed estimations we will use the Gaussian wave packet

$$\varphi(a) = e^{-a^2/2}. \quad (13)$$

We use two waves for the pulse because in the case of one wave a linear term appears in the phase of the excited atomic wave function. This term does not influence squeezing in a harmonic well, although the anharmonicity might be more important in this case. All calculations can be easily justified for the more complicated picture of one fast pulse but there will be no difference for the maximum amount of squeezing.

By "fast pulse" we mean that the atoms remain static during the pulse. Thus the pulse duration τ must be much less than the period of atomic vibrations in the well

$$\omega_h \tau \ll 1. \quad (14)$$

To make the effect strong we need to have resonant conditions between the external pulse and the stimulated oscilations of the atom dipole moment. For this purpose the frequency of the pulse must be close to the frequency of the standing wave field $\omega_* \approx \omega$.

As in Ref. [1] the synchronization of disturbed vibrations of different atoms should be provided. Consequently the distance passed by the light during the characteristic period of the atomic vibrations $2\pi\omega_h^{-1} \sim 10^{-9}s$ must be larger than the sample size. The sample size should be less than 1cm. We assume this condition to be satisfied. Hence we can neglect x-dependent part of the ϕ-function argument in Eq.(12).

Consider the phase shift caused by the action of the external field on the single atom. The effective potential acting on an atom can be represented as

$$U_{tot} = U_0 + \delta U = \frac{\hbar}{4\Delta\omega}(\Omega_R^2 + \delta\Omega_R^2\tau\delta(t))cos^2(kx), \quad (15)$$

The time dependence of the external pulse has been replaced with the δ function and the perturbation of the Rabi frequency is defined as

$$\delta\Omega_R^2 = d^2E_0^2\int_{-\infty}^{+\infty}\varphi(x)dx. \quad (16)$$

The phase shift caused by this external pulse is

$$\Phi(x) \approx \int dt\delta U(x,t) = -\frac{I_1}{I_0}\frac{\Omega_R^2}{4\Delta\omega}\tau\cdot 2k^2x^2 \quad (17)$$

where I_0 is the intensity of the standing wave and I_1 is the intensity of the pulse. The squeezing parameter ξ (6) reads

$$\xi = \frac{I_1}{I_0}\frac{\Omega_R^2}{8\Delta\omega}\tau\cdot\frac{E_R}{\hbar\omega_h}. \quad (18)$$

The validity of Eq.(6), i.e. that only a change of phase occurs in Ψ, can be checked by estimating the probability of the ground state $\rightarrow$ excited state transfer under the pulse (12)given by the matrix element

$$M(x) \sim \frac{dE_0}{\hbar}(\eta((\omega_* - \omega_0)\tau)) + \eta((\omega_0 - \omega_*)\tau))) \quad (19)$$

The requirement that the perturbation be sufficiently weak implies that this matrix element must be much less than unity. For the case of a Gaussian perturbation this condition can be rewritten as

$$\frac{dE_0\tau}{\hbar}\sqrt{2\pi}exp(-\frac{(\omega_0-\omega_*)^2\tau^2}{2}) \ll 1. \quad (20)$$

The exponentially small value of the excitation probability for the large difference between dE_0 and $\Delta\omega$ is the general consequence of the adiabatically slow change of the external field with respect to the energy difference between levels.

Thus we find that the frequency of the external pulse perturbation must be close to the frequency of the laser responsible for the formation of the optical lattice and the duration of the short pulse should exceed the inverse frequency of the transition between the ground and excited levels

$$\tau\Delta\omega \gg \hbar. \quad (21)$$

For future consideration we will simply assume that the frequency of the external pulse coincides with that of the standing wave field $\omega = \omega_*$.

Using conditions (11), (21), we can examine whether the maximum squeezing, defined the by recoil momentum of Eq.(9), can be attained, by estimating the largest pulse duration τ and pulse amplitude E_0.

We take for the radiative width

$$\gamma_R \sim 10^8 s^{-1}, \quad (22)$$

as is appropriate for alkali atoms [13]. The frequency corresponding to the recoil energy is usually about 10^2 times less than the radiative width

$$\frac{E_R}{\hbar} \sim 10^6 s^{-1}, \quad E_R \sim 10^{-9}eV.$$

It is significant that the energy $E_{rad} = \hbar\gamma_R$ defines the minimum temperature for the atoms within the commonly adopted Doppler cooling technique [12]. Recall

that to reach the minimum width in the momentum space we need to have the initial energy of the order of the quantization energy $\omega_h \sim \sqrt{UE_R}$ (see Eq.(9) and the preceding discussion). Therefore the depth U of the well in the standing wave required to get the maximum squeezing must exceed $10^{-5}eV$. This value of the well depth can be reached within existing experimental techniques (see Ref. [13]). Note that the above set of parameters corresponds to the limit of validity for small decay condition (10).

The requirements for the external pulse are of most interest. To reach maximum squeezing in accordance with Eqs. (9), (7), (6) we need to have

$$\xi^2 \sim \sqrt{\frac{U}{E_R}} \approx 10. \tag{23}$$

Using the definition of ξ Eq. (18) we get

$$\xi \sim \frac{I_1}{I_0}(\Omega_h \tau) \sim 10.$$

The latter condition implies that the amplitude of the field in the fast pulse must be at least about 3-5 times larger than in the standing wave field. This restriction does not appear to be crucial for the development of the squeezing technique. Note that the "fast" field remains adiabatically slow with respect to the motion between interatomic levels and the condition (21) as well as its possible generalizations are satisfied. Then if these conditions are satisfied the minimum momentum variance can be reached. For an alkali atom optical lattice this will give squeezing of 10 compared to 10^{-4} for the measured phonon case [1].

As regards cooling, recall that $T_{Dopp} \sim \hbar\gamma_R/k_B$ defines the minimum temperature whcih can be reached for the atoms with the most common Doppler cooling technique [12]. In the scenario proposed here the laser is abruptly switched off just when the system is in the state of maximum squeezing Eq.(9) (minimum variance) in momentum space. As shown in Eq.(9) this is the recoil momentum. The ratio of Doppler and recoil temperatures is about 100 in alkali metals. Hence our method ensures sub-doppler cooling 100 times down the Doppler limit $\sim 10^{-5}K$ during a relatively short time $10^{-8}s$. Note also that the method permits multiple repetition (laser on, laser off), and this cycling will be advantageous in reaching very low temperatures. An approach to the cooling problem in case of slower modulations of the effective Rabi frequency, based on optimal control theory, has been proposed in Ref. [11].

In summary we propose using optical lattices along with the pulse method of Garrett et al [1] to achieve remarkable squeezing of the matter field of atomic vibrations. An important result of our work is the expression for maximum squeezing (minimum variance) in Eqs.(7), (8) and especially (9). It appears this strong squeezing can be reached within existing experimental techniques to make the sub-doppler cooling of quantum gases.

An interesting conclusion can be made from our considerations not only for the optical lattices, but also for the magneto-optical traps. Namely, the minimum achievable temperature is limited by the size of the trap. Particularly, for sodium atoms the typical size of the magneto-optical trap is about $\sim 10^3$ of the resonant wavelength and therefore, the minimum temperature is $\sim 10^{-6}$ of the recoil temperature. This provides a significant reduction of the temperature.

The work was supported in part by CUNY-PSC Faculty research Award Program. A. Burin acknowledges support in part by Netherlads Organization for Scientific Research. H. Rabitz acknowledges support from National Science Foundation.

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